Assessing simplified and detailed models for predictive control of space heating in homes

Kun Zhang[1], Narges Roofigari E.[1], Humberto Quintana[1], Michaël Kummert[1]

(1) Polytechnique Montréal, Montréal, Canada

1. ABSTRACT

A model of a real system is required for predictive control to determine the best control sequence when disturbance forecasts and future system status are considered over a defined time horizon. The selected model should strike a balance between its accuracy and simplicity. This paper presents a comparison between different modeling approaches for predictive control of space heating. The case study is electric baseboard heating in homes within cold climate regions with the objective of reducing peak electricity demand (and saving costs if tariffs include a peak power charge). Detailed TRNSYS models of the selected house are developed and predictive control is implemented by using GenOpt as the optimization tool. This approach is compared with optimal predictive control algorithms based on simpler models. These models are obtained by parameter identification using data generated from the detailed TRNSYS models. Both approaches use perfect forecasts for the occupancy and the weather data in order to focus the analysis on model differences. Results show that MPC can deliver a significant reduction in power demand during on-peak periods with both modelling approaches (55% with detailed model, 33% with simplified model). The detailed model delivers significantly better savings but implies a calculation time that is more than 2 orders of magnitude higher. The potential of both approaches is discussed in the context of residential heating control to support a smart grid.

Keywords: simplified model, model predictive control, TRNSYS, GenOpt

2. INTRODUCTION

In cold climates, the peak power demand on the electrical grid is generally reached during the coldest day(s) of winter, due to the significant contribution of space heating to the peak load. In the Canadian province of Québec, the most recent historical peak was reached on January 22nd, 2014 at 7h26 AM, while the temperature was below -25 °C across most of the province (Hydro-Québec, 2014). It is estimated that residential electric heating accounts for 30 % of the grid peak electrical demand, with a market share of 70 % (Kummert, Leduc , & Moreau , 2011). Critical peak events typically represent some 50 h per year, generally occurring during a morning peak (approximately 6 AM to 9 AM) and an afternoon/evening peak (approximately 4 PM to 8 PM). Utility companies place a high value on kWh used during these critical peak events, and this paper looks at the potential of reducing the peak power demand of residential heating in a typical house during these two daily peak periods. The analysis focuses on the coldest day of the year, which is January 12th for the typical weather file used in the study.

Model predictive control has shown potential for load shifting and energy and costs savings by taking into account predictions of process states in the decision making. (Oldewurtel F. , Parisio, Jones, & Morari, 2010) and (Oldewurtel F. , et al., 2012) compared the current practice, Rule Based Control (RBC), with prediction-based approaches and confirmed additional energy savings by using a Stochastic Model Predictive Control (SMPC) as well as a predictive non-stochastic controller, so called Certainty Equivalence (CE).

MPC has been practically applied and theoretically investigated in chemical engineering since 1960s (Morari & Lee, 1999) and lately it has drawn increasing interest for supervisory control in building engineering field (Coffey, 2013).The idea of MPC proposed for building supervisory control can be dated back as early as in 1988 (Kelly, 1988) but it does not witness a steady growing research until the last decade (Prívara, et al., 2013).

The MPC research on buildings has seen a wide variety of systems including ice and building thermal mass storage (Henze, Felsmann, & Knabe, 2004) and (Henze, Le, Florita, & Felsmann, 2006), window operation for mixed natural and mechanical ventilation in an office building (May-Ostendorp P. , Henze, Corbin, Rajagopalan, & Felsmann, 2011) and (May-Ostendorp P. , Henze, Rajagopalan, & Corbin, 2013), VAV control (Wang & Jin, 2000) and (Nassif, Kajl, & Sabourin, Simplified model-based optimal control of VAV air conditioning system, 2005a), (Nassif, Stainslaw , & Sabourin, 2005b) and other systems proposed in the Model Predictive Control in Buildings workshop in Canada (IBPSA-Canada, 2011) such as thermally activated building system with ground coupled heat pump (Verhelst, Sourbron, Antonov, & Helsen , 2011), chiller and cooling tower system (Ma & Borrelli, 2011), and so on.

Among the research issues in MPC on buildings, the foremost is the choice of building models, which determines the effectiveness and efficiency of control strategies. The main building models utilized by researchers can be divided into three categories: physical models built by building energy software programs (e.g. EnergyPlus, TRNSYS), grey box (e.g. RC network) and data-driven black box models. Because of the complexity and high-computation requirement of the physical models, they are mostly used with other optimization tools (e.g. GenOpt) for offline predictive control (Coffey, 2013) and control rules extraction (May-Ostendorp P. , Henze, Rajagopalan, & Corbin, 2013). Simpler models can be used for real-time online MPC due to their better suitability for online parameter identification and their lower computational requirements.

A simplified model for the transient heat transfer through a multilayer structure can be developed based on simulating the heat transfer by the concept of lumped resistance and capacitances. There exist two general approaches for obtaining proper values for this network of lumped resistance and capacitances. One approach is based on physical characteristics of building elements. This approach requires knowledge about details of building elements including zones specifications, constructions and materials. Sturzenegger et al. in (Sturzenegger, Gyalistras, Semeraro, Morari, & Smith, 2014) developed a Toolbox for generation of bi-linear resistance-capacitance models based on this approach. Lehmann et al. in (Lehmann, Gyalistras, Gwerder, Wirth, & Carl, 2013) also proposed an intermediate-complexity (12^{th} order) bilinear model for a single room. The other approach is based on parameter identification. This approach requires sufficient excitation conditions that yield determining a precise model. This approach can be performed in time or frequency domain. Also there exist different mechanisms for identifying the parameters. Time domain input/output data-based techniques include extended Kalman filters (Huchuk, A. Cruickshank, O'Brien, & Gunay, 2014), maximum likelihood, prediction error minimization and subspace system identification (Candanedo, Dehkordi, & Lopez, 2014). This approximation results in a simplified model which in general is not adaptable to parameter variations. The model may or may not describe long-term dynamics depending on the number of time constants of the corresponding RC network. Madson and Holst in (Madson & Holst, 1995) suggested using a two time constant model for a single-story building. They utilized maximum likelihood method for identifying the model parameters. (Wang & Xu, 2006) combined physics law approximation with parameter identification based on operation data to obtain a simplified model of a thermal zone in a building yielding a multiple time constant model that takes into account internal mass and multilayer external wall/roof thermal dynamics. They used a genetic algorithm to identify the parameters corresponding to the internal mass.

3. OBJECTIVE AND METHODOLOGY

The objective of this work is to contrast two modeling approaches in assessing the potential of MPC to reduce the power demand associated with space heating during critical grid peak event. The first method uses a detailed (physical) building model implemented in TRNSYS and a generic optimization tool (GenOpt) to perform control optimization. The second approach uses a simple RC network to model the same building, together with a parameter identification method and an optimization process implemented in Matlab. Both methods are compared using a simulation environment based on TRNSYS and combining GenOpt to TRNSYS (detailed model) or Matlab to TRNSYS (RC model).

3.1 Methodology

The following steps are the common path for the implementation of the two approaches. More details can be found in the corresponding sub-sections.

- Define the main assumptions (optimization problem) for this study
- Implement a TRNSYS model of the selected building to act as a reference that will be used to assess the different control strategies
- Define a cost function that includes on-peak/off-peak electricity cost and penalties for constraints violation
- Implement the MPC approach with both models
    - Couple GenOpt and TRNSYS and compare different optimization strategies based on the detailed TRNSYS model
    - Develop a simplified RC model in Matlab, perform parameter identification, couple Matlab to TRNSYS and implement the optimal MPC strategy in Matlab.
- Assess both control strategies using the detailed TRNSYS model – this means that the first control strategy will have no modeling error, as the same model is used both to develop the control strategy and to assess its performance

3.1.1 Main assumptions – optimization problem

The considered problem is to reduce the electrical demand for heating in a typical house during the two periods representing critical conditions for the electrical grid. For the sake of this study, the critical period is defined as 5:30 AM to 9:30 AM and 4:30 PM to 8:30 PM. Electricity is not considered to be free outside of these on-peak periods but its value is significantly reduced – the context is to assess the potential of peak savings for a utility, not to optimize a customer's electrical bill with a realistic on-peak / off-peak tariff.

The control system is allowed to modify the setpoints in the basement and the living area of the house, but not in the bedrooms (which are also electrically heated). It is assumed that the thermostat setpoint can be adjusted between 20 °C and 23 °C when the rooms are occupied; between 18 °C and 23 °C when they are not occupied. Occupancy in the living area and basement is assumed to be the same, during a morning period (6:30 AM to 8:00 AM after the occupant wake up and before they leave the house) and an evening period (4:30 PM to 10 PM after the occupants return from school/work and before they go to their bedrooms).

Figure shows the on-peak periods as yellow areas and the occupancy periods as blue rectangles. The allowable range for the thermostat setpoints is also shown by dotted lines.

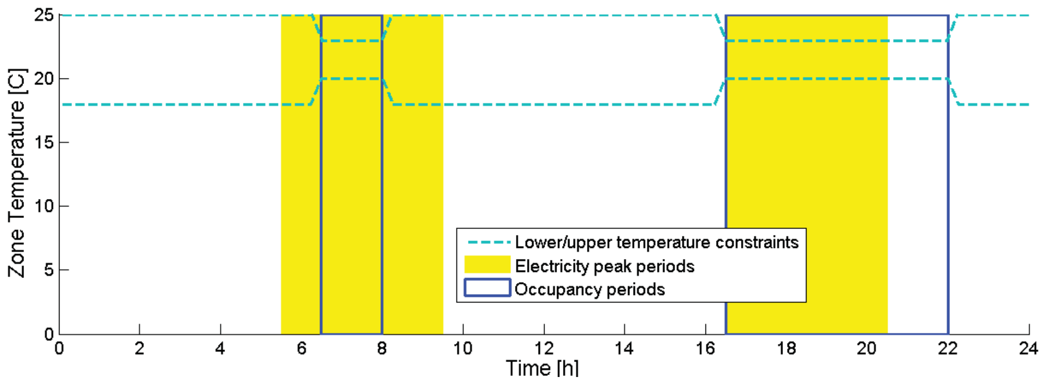

Figure 1: Occupancy, on-peak periods and allowable range for thermostat setpoint

Only one day is considered in the optimization study, January 12th. That day is the coldest day in the typical weather year (CWEC) for Ottawa. The simulation is run form January 1st to January 12th to allow for building pre-conditioning, as the multizone building model in TRNSYS (Type 56) is initialized with unrealistic temperature profiles (uniform temperature across zones and building envelope).

Uncontrolled disturbances such as weather variables (temperature, solar radiation, etc.) and internal gains (from occupants, lighting and appliances) are considered to be known to the optimization process (perfect forecasts). This will provide a higher bound for the performance of both MPC algorithms and isolate the differences attributable to different models from other influences.

3.1.2 Selected house

In 1998, the Canadian Centre of Housing Technology (CCHT) built twin houses for research purposes according to the R-2000 standard (increased energy performance and air tightness). The houses (Figure 2) are typical North-American wood-frame buildings with brick facing, and have five main zones (basement, two floors, garage and attic) and the liveable area is approximately 210 m^2. Home automation systems simulate occupancy by activating appliances, lights, water valves and incandescent bulbs (for internal gains due to humans). Measures are collected by 23 meters and more than 250 sensors providing 12000 readings every 24 hours (Swinton, et al., 2001).

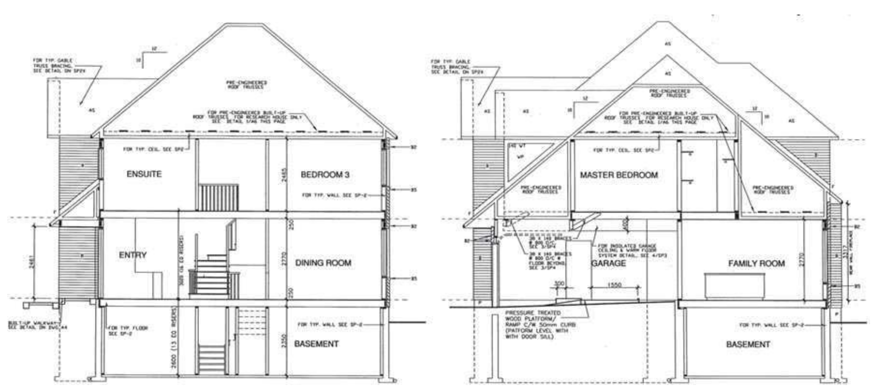

Figure 2: CCHT house drawings

3.1.3 TRNSYS model

The TRNSYS model of the CCHT house has five thermal zones (first floor, second floor, basement, garage, and attic) and schedules for thermal internal gains (people, appliances and lighting). The basement and first floor are assumed to be living spaces, while the second floor is a sleeping space with a different occupancy profile. Other zones are not occupied. No detailed HVAC system is modeled and the idealized convective heating available in the TRNSYS multizone building model (Type 56) is used, as it is a reasonably accurate representation of electric baseboard heating commonly used in Québec. The maximum heating power in living room, sleeping room and basement is set to 7.5kW, 5kw and 5kW to represent the capacity of installed electric baseboards.

Type 56, the multizone building model in TRNSYS, is the main element of the system model. This component (TRNSYS "Type") has all the details about house's geometry and materials, and optional inputs for external heating/cooling and internal gains. Other elements of the TRNSYS model define the shading and the *basement-ground coupling model* (Type 1244). Infiltration is set to a constant value of 0.075 ACH.

3.1.4 Cost function

The cost function J , in equation (1), represents a measure of the power demand for the on-peak and off-peak hours but not the real cost of electricity or power.

$$J = \sum_{i=1}^{96} \left\{ R_i U_i + \mu \left\{ \max\left(0, T_i - T_{u_i}\right) + \max\left(0, T_{l_i} - T_i\right) \right\} \right\} \tag{1}$$

In this cost function the first term represents the heat power demand costs, where U_i denotes the heat power at time i. R_i denotes the heat power price. As the optimization objective is to minimize power demand during peak hours, a relatively larger weighting factor is assigned for the on-peak power by setting the price to be 100 times of that in off-peak periods.

The second term in this cost function is a penalty function that guarantees the desired temperature comfort levels. i.e., $T_{l_i} \le T_i \le T_{u_i}$, where T_i denotes the temperature and T_{l_i} and T_{u_i} represent the lower and upper bounds at time i respectively. μ denotes the weighting factor for this penalty function. Its value is selected to be 10000 by trial and error to implement a soft constraint on the room temperature without compromising numerical stability. The penalty function is evaluated as zero when the room temperature is between the defined intervals but is much larger when the temperature does not obey the comfort limits.

The actual cost function implemented in the two MPC approaches is slightly different (see details in the corresponding sections below), but the performance presented in the Results section is always assessed using the cost function described here above.

4. IMPLEMENTATION

4.1 Optimization with GenOpt

GenOpt is a generic program developed mainly for building system optimization with an extended library of optimization algorithms. It can be used with any text-based simulation program (Wetter, 2001).

As illustrated above, TRNSYS is used for the building simulation. Before launching the optimization in GenOpt, templates of input and output files have been created in TRNSYS. In each optimization, GenOpt updates the variables, i.e. set points in this case, in the templates.

GenOpt then searches for the minimal cost function value among all the optimization results. In this study, the "Hybrid Generalized Pattern Search Algorithm with Particle Swarm Optimization" Algorithm is employed (Wetter, 2011).

The cost function is calculated inside of TRNSYS, as well as penalty function and all other constraints, so that GenOpt only needs to read the designated parameters in corresponding files.

4.2 Optimization with Matlab

In this section, the procedure for developing an online MPC-based heating control is presented. The following subsection derives the simplified model that is required for this control approach.

4.2.1 Simplified model and parameter identification

The concept of thermal zone can be utilized to describe the heat flux in a building. Once a thermal zone is specified, a lumped RC network can describe the zone temperature at each time. The model may or may not describe long-term dynamics depending on the number of time constants of the corresponding RC network. In this work we use a network consisting of six resistances and three capacitances as shown in Figure 3. In this figure, ϕ_s, ϕ_l and ϕ_b denote the overall heat that is being injected in the zones. This heat includes the (controlled) heating power, internal heat gains and solar radiation.

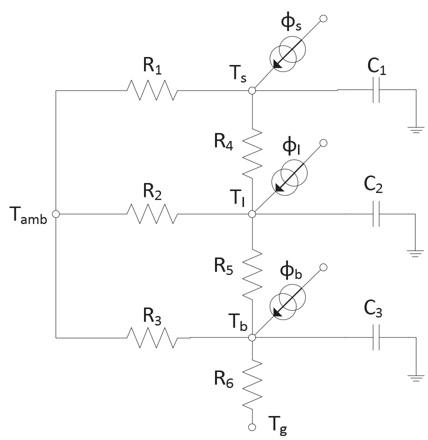

Figure 3: RC model schematic

Therefore, the following model describes thermal dynamics in the model.

$$\dot{x} = A_c\, x + B_c U + E_c W \qquad (2)$$

Where the vector $x = [T_s \quad T_l \quad T_b]^T$ denotes the matrix of system states consisting the sleeping room, living room and basement temperatures. $U = [U_s \quad U_l \quad U_b]^T$ denotes the control inputs which are heat addition rates (heating power) to each zone, and $W = [\phi_{IG\_s} \quad \phi_{IG\_l} \quad \phi_{IG\_b} \quad \phi_{sol} \quad T_{amb} \quad T_g]$ represents the disturbance inputs where ϕ_{IG_s}, ϕ_{IG_l} and ϕ_{IG_b} denote the internal heat gains corresponding to each zone, ϕ_{sol} denotes the solar radiation incident on the South façade (used as a proxy for the total solar gains), T_{amb} denotes the ambient temperature and T_g denotes the ground temperature. The triple (A_c, B_c, E_c) is obtained as follows and consists of parameters that will be identified.

$$A_c=\begin{bmatrix} -\left(\frac{1}{R_4C_1}+\frac{1}{R_1C_1}\right) & \frac{1}{R_4C_1} & 0 \\ \frac{1}{R_4C_2} & -\left(\frac{1}{R_4C_2}+\frac{1}{R_2C_2}+\frac{1}{R_5C_2}\right) & \frac{1}{R_5C_2} \\ 0 & \frac{1}{R_5C_3} & -\left(\frac{1}{R_3C_3}+\frac{1}{R_5C_3}+\frac{1}{R_6C_3}\right) \end{bmatrix}, B_c=\begin{bmatrix} \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & \frac{1}{C_3} \end{bmatrix}, E_c=\begin{bmatrix} \frac{1}{C_1} & 0 & 0 & \frac{\alpha_1}{C_1} & \frac{1}{R_1C_1} & 0 \\ 0 & \frac{1}{C_2} & 0 & \frac{\alpha_2}{C_2} & \frac{1}{R_2C_2} & 0 \\ 0 & 0 & \frac{1}{C_3} & \frac{\alpha_3}{C_3} & \frac{1}{R_3C_3} & \frac{1}{R_6C_3} \end{bmatrix} \quad (3)$$

Where α_1, α_2 and α_3 denote multiplication coefficients applied to the solar radiation for each thermal zone.

We used prediction error minimization for the purpose of identifying the twelve parameters. As such, we consider the discrete time form of the equation (2) as follows:

$$x(k+1)=Ax(k)+BU(k)+EW(k) \quad (4)$$

Where the triple $(A,B,E)\in\Re^{3\times3\times6}$ denotes the discretized form of (A_c,B_c,E_c) by using zero-order hold method.

We assume the availability of the information about the state vector, control inputs and the disturbance inputs and estimate the parameter values that result in the minimal error between the predictions and true states by solving the following problem for the parameter identification of the three-zone building:

Given $x(k)$, $U(k)$ and $W(k)$ for $k=0,1,\ldots,N$, find the matrices A, B and E that minimize the error function E:

$$E=\sum_{k=0}^{N-1}\left(\hat{x}(k+1)-x(k+1)\right)^{\mathrm{T}}\left(\hat{x}(k+1)-x(k+1)\right) \quad (5)$$

Where $\hat{x}(k)$ denotes the model state at time k and depends on the 12 parameters, namely R_1, R_2, R_3, R_4, R_5, R_6, C_1, C_2, C_3, α_1, α_2, α_3, as follows:

$$\hat{x}(k+1)=Ax(k)+BU(k)+EW(k) \qquad \text{for } k=0,1,\ldots,N-1 \quad (6)$$

In this identification problem, N denotes the number of training samples and therefore the period of time selected for training will be $N\times\frac{1}{f_s}$ hours where f_s denotes the sampling frequency ($\left[1/hour\right]$). In the next section, we use the identified building thermal model for the purpose of centralized control of zone temperature setpoints.

The identification and validation algorithms can be summarized by the following steps:

- Identification
    a) [Training Step]: The optimal control input (Power heat) $U(k)=\left[U_s(k)\quad U_l(k)\quad U_b(k)\right]^{\mathrm{T}}$ corresponding to the obtained temperature setpoints from the GenOpt optimization are considered as the control input during the first 12 days of January. This choice ensures that the generated input/output training data provides enough excitation. The corresponding outputs, i.e., the zone temperatures $T_s(k)$, $T_l(k)$ and $T_b(k)$ for $k=0,1,\ldots,12\times24\times f_s-1$ are "measured" (received from the TRNSYS simulation) and the disturbance inputs $W(k)$ for $k=0,1,\ldots,12\times24\times f_s-1$ are collected. Note that due to the modelling structure, all

of the states of the system are measurable and therefore there is no need to observe the states.

b) The one-step ahead predicted outputs, i.e., $\hat{x}(k+1)=\left[\hat{T}_s(k+1) \quad \hat{T}_l(k+1) \quad \hat{T}_b(k+1)\right]^{\mathrm{T}}$ for $k=0,1,\ldots,12\times24\times f_s-1$ are formulated based on current system states $x(k)=\left[T_s(k) \quad T_l(k) \quad T_b(k)\right]^{\mathrm{T}}$, current control input $U(k) = \left[U_s(k) \quad U_l(k) \quad U_b(k)\right]^{\mathrm{T}}$, current disturbance input $W(k)$ for $k=0,1,\ldots,12\times24\times f_s-1$, and the unknown model, i.e., (A,B,E) and by using equation (6).

c) The prediction error minimization problem is solved by using interior point algorithm, yielding (A,B,E).

d) Finally, the parameters are identified as follows:
   - The diagonal values of the matrix B give the capacitance values (C_1, C_2, C_3).

   Once the capacitance values are determined,
   - The 4th column of the matrix E gives the solar radiation coefficient values (α_1, α_2, α_3).
   - The 5th and the 6th columns of the matrix E give R_1, R_2, R_3, R_6 values.
   - $A_{1,2}$ and $A_{2,3}$ give R_4 and R_5.

- Validation

a) The optimal control input (Power heat) $U(k) = \left[U_s(k) \quad U_l(k) \quad U_b(k)\right]^{\mathrm{T}}$ corresponding to the obtained temperature set-points from the GenOpt optimization are considered as the control input during 12th day of January. The corresponding outputs, i.e., the zone temperatures $x(k+1)=\left[T_s(k+1) \quad T_l(k+1) \quad T_b(k+1)\right]^{\mathrm{T}}$ for $k=11\times24\times f_s,\ldots,12\times24\times f_s-1$ are "measured" (received from the TRNSYS simulation) and the disturbance input $W(k)$ for $k=11\times24\times f_s,\ldots,12\times24\times f_s-1$ are collected. As mentioned previously, all of the states of the system are measurable and therefore there is no need to observe the states.

b) The one-step ahead predicted outputs, i.e., $\hat{x}(k+1)=\left[\hat{T}_s(k+1) \quad \hat{T}_l(k+1) \quad \hat{T}_b(k+1)\right]^{\mathrm{T}}$ for $k=11\times24\times f_s,\ldots,12\times24\times f_s-1$ are calculated based on current system states $x(k)=\left[T_s(k) \quad T_l(k) \quad T_b(k)\right]^{\mathrm{T}}$, current control input $U(k) = \left[U_s(k) \quad U_l(k) \quad U_b(k)\right]^{\mathrm{T}}$, current disturbance input $W(k)$ for $k=11\times24\times f_s,\ldots,12\times24\times f_s-1$, and the previously determined model, i.e., (A,B,E) and by using equation (6).

c) The root mean square deviation (RMSD) between the measured outputs and the one-step ahead predicted outputs corresponding to all three zone temperatures during 12th day of January are calculated and obtained as 0.07, 0.4 and 0.5 °C for sleeping room, living room and basement respectively and the corresponding RMSD for the 24 hours-step ahead predictions are obtained as 0.7, 1.15 and 2.05 °C for sleeping room, living room and basement respectively.

4.2.2 MPC Approach

This section presents the optimal set-point solution to the MPC problem for the building. In this structure the following dynamic thermal equations that were developed in Section 4.2.1, govern the control design through performing the predictions based on the control input information.

$$x(k+1) = Ax(k) + BU(k) + EW(k) \tag{7}$$

We are now in position to state our MPC problem.

At any time t_k, $k \in \{0,1,2,\ldots\}$, given $x(k)$ and $\{W(k), W(k+1), \ldots, W(k+P-1)\}$, find the input sequence $\{U(k\,|\,k), U(k+1\,|\,k), \ldots, U(k+P-1\,|\,k)\} \in \Re^3$, that minimizes the cost function $J(k)$:

$$J(k) = \sum_{i=0}^{P-1} U^{\mathrm{T}}(k+i\,|\,k) R(k+i) U(k+i\,|\,k) \tag{8}$$

Where the prediction equations are as follows

$$x(k+i+1\,|\,k) = Ax(k+i\,|\,k) + BU(k+i\,|\,k) + EW(k+i\,|\,k) \quad \text{for } i = 0,1,\ldots,P-1 \tag{9}$$

and $x(k\,|\,k) = x(k)$

Moreover, the constraints of the problem are given by

$$S(k+i)U(k+i\,|\,k) \le s(k+i) \qquad \text{for } i = 0,1,\ldots,P-1 \tag{10}$$

$$G(k+i)x(k+i+1\,|\,k) \le g(k+i) \qquad \text{for } i = 0,1,\ldots,P-1 \tag{11}$$

$$G_{eq}(k+i)x(k+i+1\,|\,k) = g_{eq}(k+i) \qquad \text{for } i = 0,1,\ldots,P-1 \tag{12}$$

In equation (8), the prediction horizon is represented by P and $\{U(k\,|\,k), U(k+1\,|\,k), \ldots, U(k+P-1)\}$ denotes the set of designed control inputs that minimize the objective function. $R(k+i) \in \Re^{3\times 3}$ denotes a positive definite matrix representing the input penalty matrix corresponding to the time instant $k+i$ for $i = 0,1,\ldots,P-1$. In equation (9) (A, B, E) represents the linear state space model as previously given in equation (7). Equation (10) represents the constraints on inputs. Equation (11) represents the set of state constraints at each time instant $k+i+1$ for $i = 0,1,\ldots,P-1$. These constraints represent the lower and upper limits of temperature, i.e., the temperature comfort zone. Finally, equation (12) represents a set of equality constraints on zones temperatures, namely, the zones that are assigned to have equal temperatures and the zones that are assigned to have a certain temperature value. The constraints stated in equation (10) (11) and (12) can be translated to constraints on control input by using equation (9) and represented in the form of linear constraints as follows

$$\begin{cases} S(k+i)U(k+i\,|\,k) \le s(k+i) \\ G(k+i)\big(Ax(k+i\,|\,k) + BU(k+i\,|\,k) + EW(k+i\,|\,k)\big) \le g(k+i) \\ G_{eq}(k+i)\big(Ax(k+i\,|\,k) + BU(k+i\,|\,k) + EW(k+i\,|\,k)\big) = g_{eq}(k+i) \end{cases} \quad \text{for } i = 0,1,\ldots,P-1 \tag{13}$$

The optimization algorithm can be summarized by the following steps:

- At any time t_k, $k \in \{0,1,2,\ldots\}$:
  a) $T_s(k)$, $T_l(k)$ and $T_b(k)$ are "measured" (received from the TRNSYS simulation).
  b) The MPC optimization problem is solved by using active set algorithm, yielding $U^*(k+i\,|\,k)$, for $i = 0,1,\ldots,P-1$.

c) The zone temperatures $T^*{}_s(k)$, $T^*{}_l(k)$ and $T^*{}_b(k)$ corresponding to the optimal control signals are obtained from the internal model provided in equation (9).
d) $T^*(k) = \lfloor T^*{}_s(k) \quad T^*{}_l(k) \quad T^*{}_b(k) \rfloor$ are sent to the TRNSYS model to be used as thermostat set-points ($T^*{}_s(k)$ is actually ignored in this study)
e) Only the first control signal, i.e., the heating power, is applied and the optimization process is started again at the next time step (receding horizon).

5. RESULTS

All the results presented in this section are for January 12, as an example of a very cold day that would cause a critical peak event on the electrical grid. From the utility's point of view the objective is to reduce the peak power demand of a large set of houses where demand-shifting strategies would be implemented. The main criterion used to assess the performance of different control strategies has therefore been selected as the average power used during the on-peak period, rather than the absolute peak power for one particular house. The maximum power during the on-peak period is also reported in the results.

5.1 Base cases

Two base cases are considered for comparison with MPC results: constant setpoint and night setback cases, as shown in Figure 4 and Figure 5. The heating power in the sleeping zone is reported only for reference, because it will not be affected by predictive control strategies.

The constant setpoint profile in Figure 4 results in a significant electrical demand during the on-peak periods; while the demand is actually lower during most of the day (January 12th is a sunny day, which is often the case for extremely cold days in Québec). The average power demand during on-peak periods is 6 kW.

The night setback setpoint profile shown in Figure 5 leads to a large increase of the heating power at the end of the 2 setback periods, which were set between 7:45 and 16:00, and between 21:30 and 6:00 (the setback periods are chosen by trial and error so that the temperature can reach the thermal comfort requirement just when the living room is occupied). This results in a higher power demand at the beginning of the morning on-peak period but since the setback period starts within the on-peak period, the overall performance (indicated by the average electrical power demand during the on-peak periods) is marginally worse than for the constant setpoint scenario, at 6.6 kW.

In both Figure 4 and Figure 5, we can see that the heating power drops suddenly at around 19 h because dishwasher (1.7 kW) and dryer (8.1 kW) are turned on at this moment. Other drops are due to one or more zones overheating due to internal gains or solar gains.

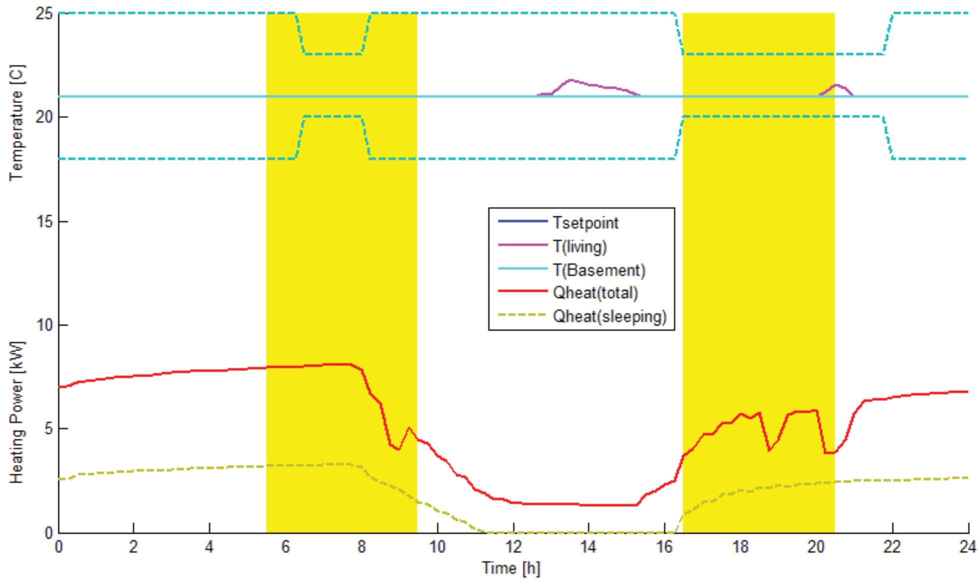

Figure 4: Constant Setpoint

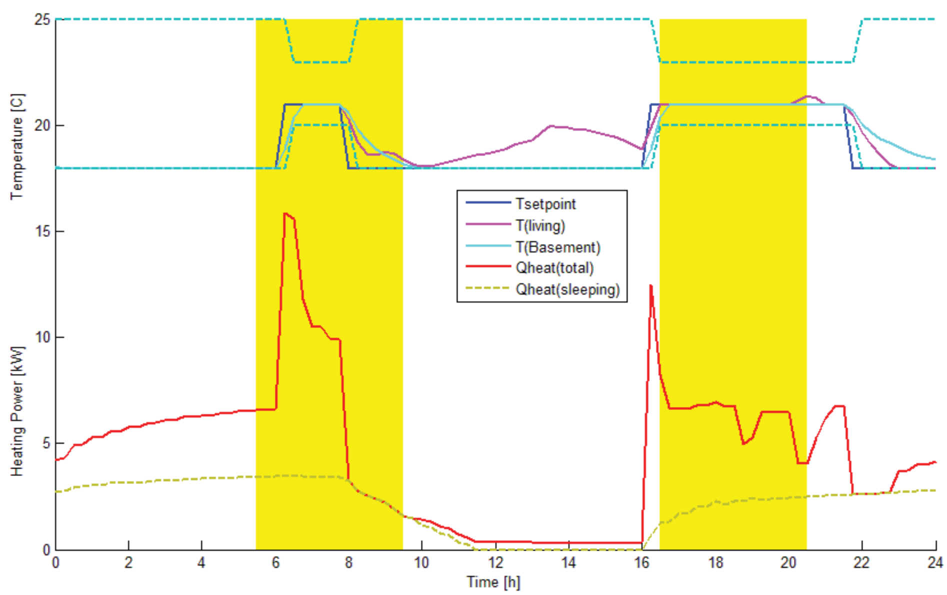

Figure 5: Night Setback

5.2 GenOpt + TRNSYS MPC

The setpoint profile obtained by MPC with GenOpt is shown in Figure 6. The optimal control strategy reduces the heating setpoint during unoccupied periods (even though the "cost" associated with electricity within these periods is very low), and ramps up the setpoint to the maximum allowed value approximately 3 h before the on-peak periods so that the building is preheated (the preheating is shorter in the afternoon). The building is then left free-floating until it cools down to the lower setpoint limit.

While the general shape of the setpoint profile corresponds to what was expected, the jagged profile is somewhat surprising. The sudden changes in the setpoint can be attributed to two phenomena. First, there is a lack of feedback to the optimization process in case of a sudden drop: once the setpoint drops faster than the rate at which the building cools down naturally, there is no difference in the cost function until the setpoint reaches some constraints. Second, GenOpt algorithms are sensitive to numerical noise in the cost function and building simulation programs often results in noisy numerical results. The optimization process was also found to be sensitive to initial values, as will be shown below in Section 5.4.

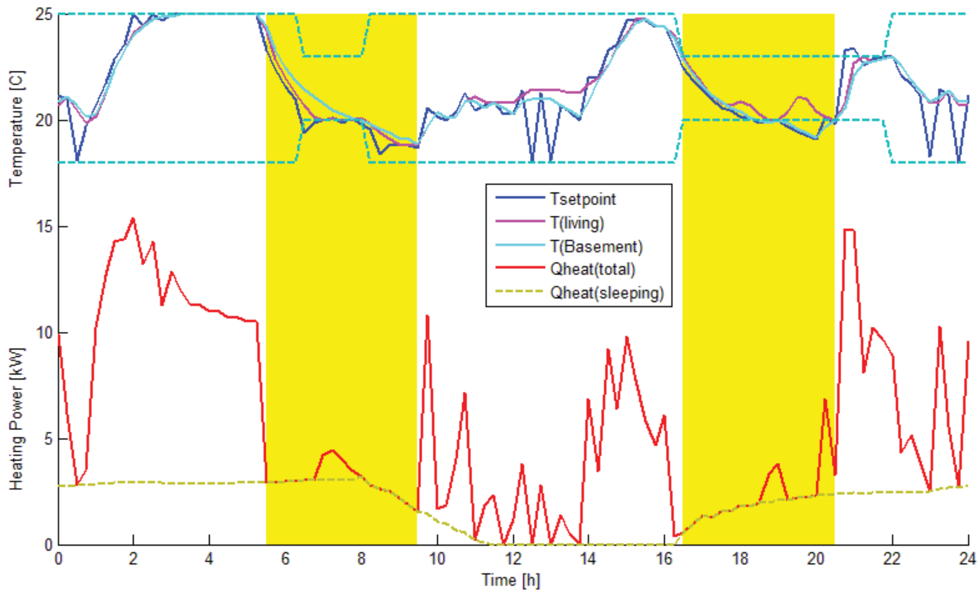

Figure 6: MPC with GenOpt

The average power demand during on-peak periods is 2.6 kW, which is a significant reduction (over 50 %) compared to the base cases. The computing time is very significant, with a 12-day simulation (with a 1-day optimization) taking more than 16 h of computing time on an i7-4770 CPU (3.5 GHz) – the same simulation with a constant setpoint or night setback profile takes about 20 seconds.

5.3 Matlab (RC-model) MPC

The internal model required by the online MPC approach was obtained using first 12 days of January as the training samples. For the purpose of online control with MPC, the optimization problem was solved by using FMINCON function in MATLAB. The prediction horizon is set to 24 hours as in the case of the GenOpt optimization. However, other tests have shown that similar results are obtained for any prediction horizon longer than 4 hours. A receding horizon is applied and the process is repeated at every time step. It can be clearly observed from Figure 7 that the online controller decides to preheat the living room around 1:15 AM, a long time before the peak period starts. The strategy then differs from the GenOpt-TRNSYS results in that some heating is applied during the first on-peak period. This results in a lower cooling down rate that seems to miss the minimum allowable value at the end of the morning occupancy period. Preheating is then used again before the second on-peak period, although not using the maximum available power. Again, the profile is sometimes jagged, which can be partly attributed to the same reasons as for the GenOpt-TRNSYS results. In addition, the model used internally by the optimization algorithm is not 100 % accurate (as shown by RMSD values in section 4.2.1), which leads to corrections at each time step given the receding horizon method. The MPC results are also sensitive to initial values and to cost function parameters, as discussed below.

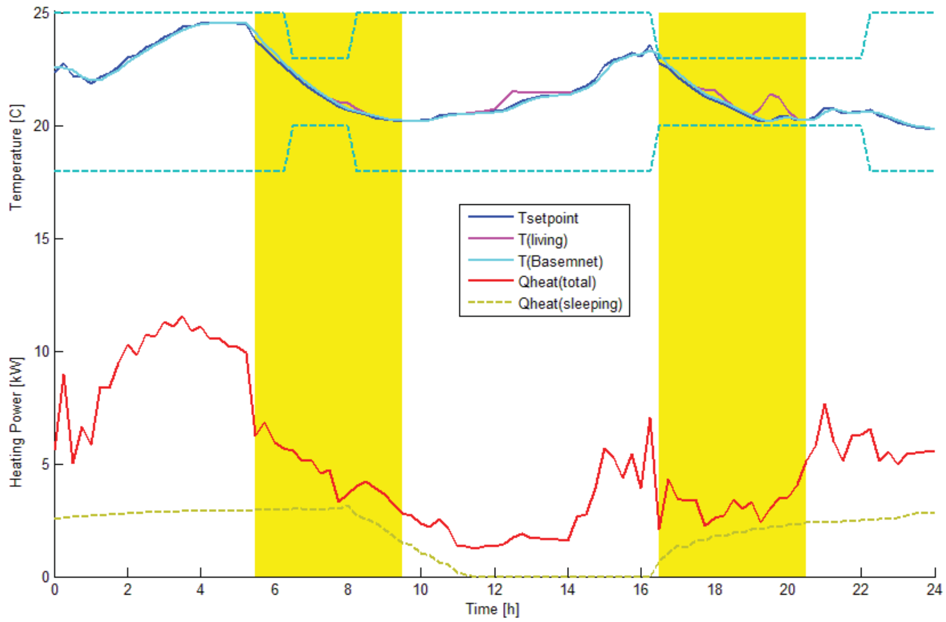

Figure 7: MPC with MATLAB

The average power demand during on-peak periods is about 4 kW, which is not as impressive as the GenOpt-TRNSYS results but is still a significant reduction (33 %) compared to the base cases. The computing time is very acceptable, with the 12-day simulation taking about 4 minutes (12 times longer than the constant profile, but about 250 times faster than TRNSYS-GenOpt).

5.4 Sensitivity of the optimization processes

During the development of the optimization methodology, several different initial conditions were tested. The results presented above for the GenOpt-TRNSYS optimization rely on initial values that are based on a previous study (Kummert et al, 2011).

If initial values are set to 21 °C for the whole day, the optimization process reaches a very different solution with large oscillations shown in Figure 8. Even though the difference in cost function (power used during the on-peak periods) is only marginally affected, the solution is clearly less desirable than the one obtained with “informed” initial values. The computational time is also affected (more than doubled).

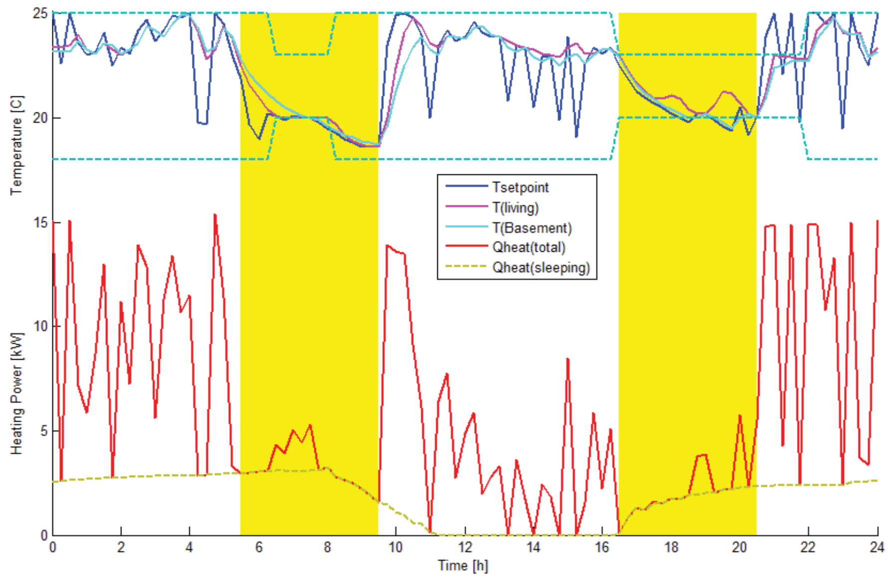

Figure 8: GenOpt-TRNSYS MPC with poor initial values

Very large oscillations can be observed on setpoint temperatures and heating power. This can be partly explained by the fact that the exact value of the setpoint has no impact on the building behavior once it is above the temperature that could be reached with full power or once it is below the temperature that would be reached in free-floating, without heating. So there is no impact on the cost function when GenOpt tries very different values of the setpoint. To illustrate this, consider the situation at 11 AM. GenOpt reduces the setpoint drastically, which results in no heating power being required. The building reaches a temperature close to 23.5 °C in free-floating, while the setpoint is at 20 °C. For that particular time step, the results and the cost function would be exactly the same for any setpoint below 22.5 °C. So the value of 20 °C is somewhat arbitrary and affected by numerical artifacts. One possible workaround to avoid such behavior would be to impose an additional penalty on rapid changes in the setpoint.

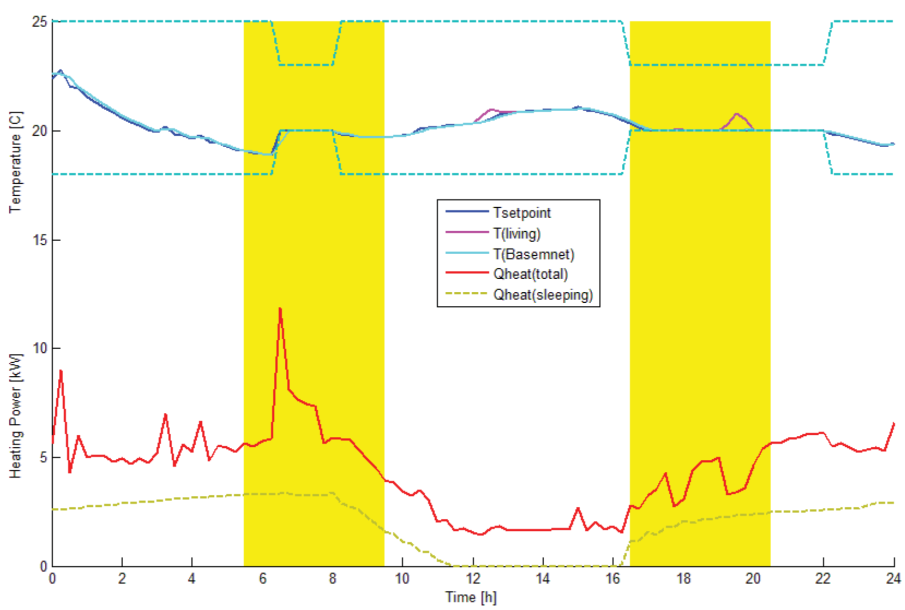

Figure 9: MPC with MATLAB Sensitiviy with off-peak cost of 0.1

The Matlab (RC-model) MPC is also sensitive to optimization parameters. The result obtained by assigning different values for the heating power matrix, i.e., the off-peak cost of 0.1 instead of 0.01 is depicted in Figure 9. It can be concluded that if we consider higher cost for the heat power during the off-peak period, no preheating is applied in the morning, nor in the afternoon. While it can be expected that preheating will become less interesting as the cost of off-peak power rises, the ratio of 0.1 is still sufficiently high that some level of preheating would be expected to lower the overall cost.

5.5 Adding heuristics to the GenOpt-TRNSYS optimization

Simplified setpoint profiles were shown to decrease the computational requirements of complex optimization processes while delivering almost the same cost savings, e.g. in (Braun, 2006). Figure 10 illustrates the "Jump and Trim" profile, where 5 setpoint values are optimized (versus 96 for a daily optimization problem with a 15-min time step). This method delivers almost the same savings (48.6% as shown in Table 1) as the GenOpt-TRNSYS optimization, with a much reduced computational time (20 minutes vs. 16 hours).

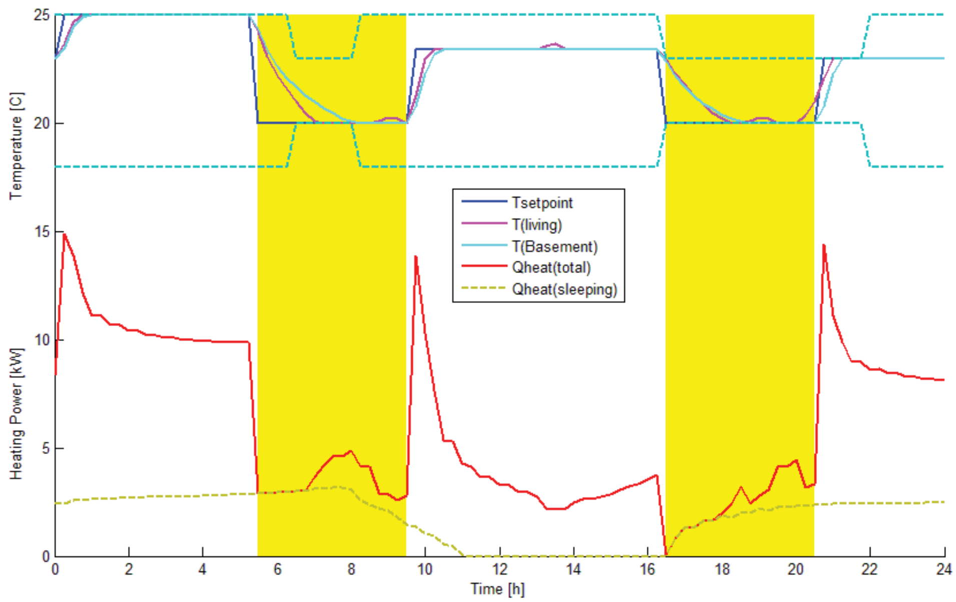

Figure 10: Jump and Trim profile

The "Linear Setpoint" profile differs from Jump and trim in that the temperature drops linearly during each 4h stretch instead of one time step as shown in Figure 11; while "Exponential Setpoint Profile" lets the setpoint decrease exponentially, with two different time constants during the two on-peak periods. The exponential profile delivers 49.4% savings while the linear profile 37.9%; but both methods show about the same reduction in computational time (down to 20 minutes).

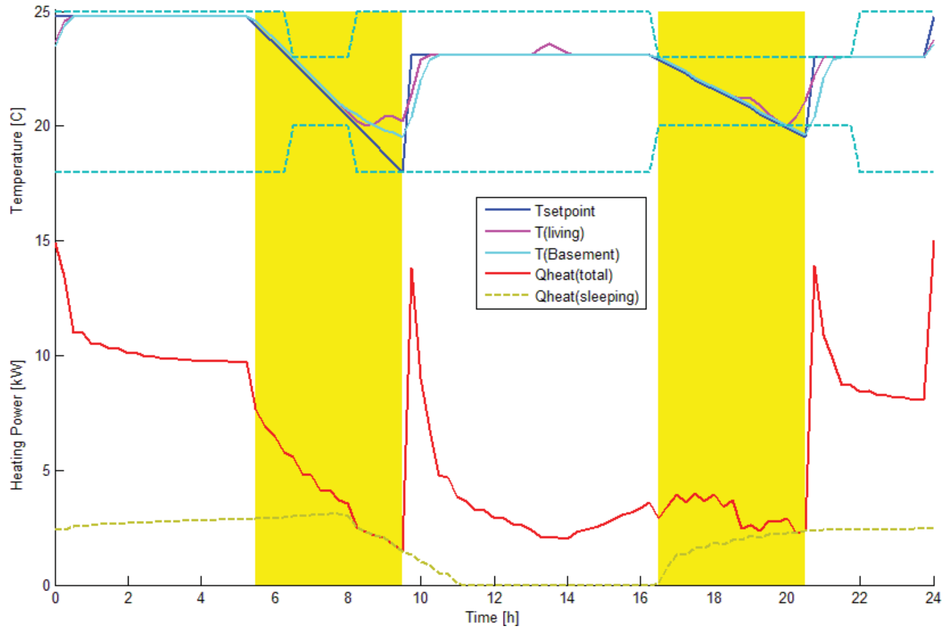

Figure 11: Linear Setpoint Profile

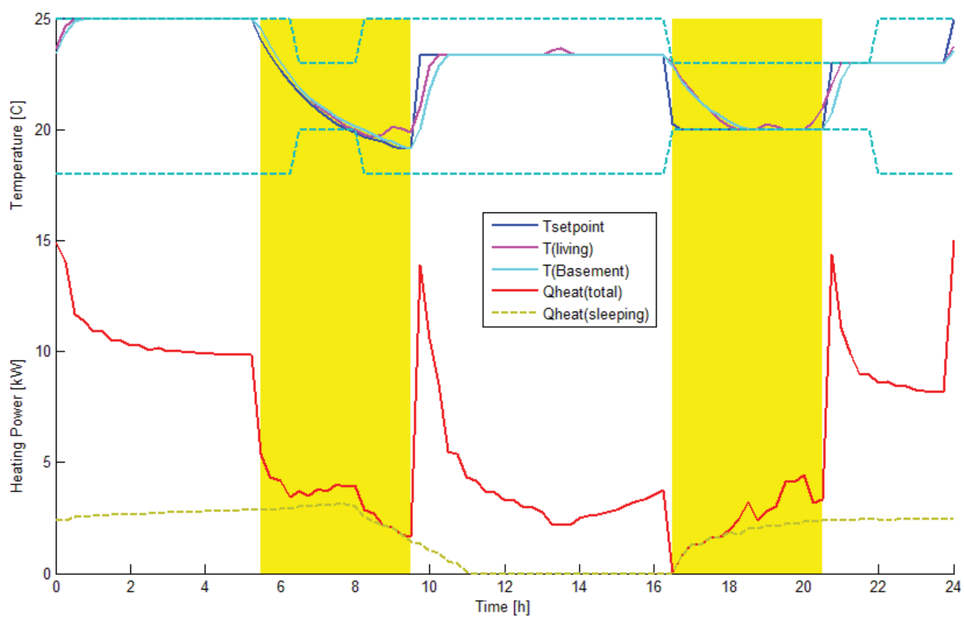

Figure 12: Exponential Setpoint Profile

Finally, the last heuristic method considered is to find the lowest power demand during the peak time, which might be interesting in the perspective of a utility company allowing a given power usage to selected customers during critical peak events. Figure 13 illustrates the results of this approach. The average power usage during on-peak periods is slightly higher than for the other optimization results, but the maximum power requested at any 15-min time step by the house is the lowest of all, at 4 kW (vs. 5 to 8 kW for the other results).

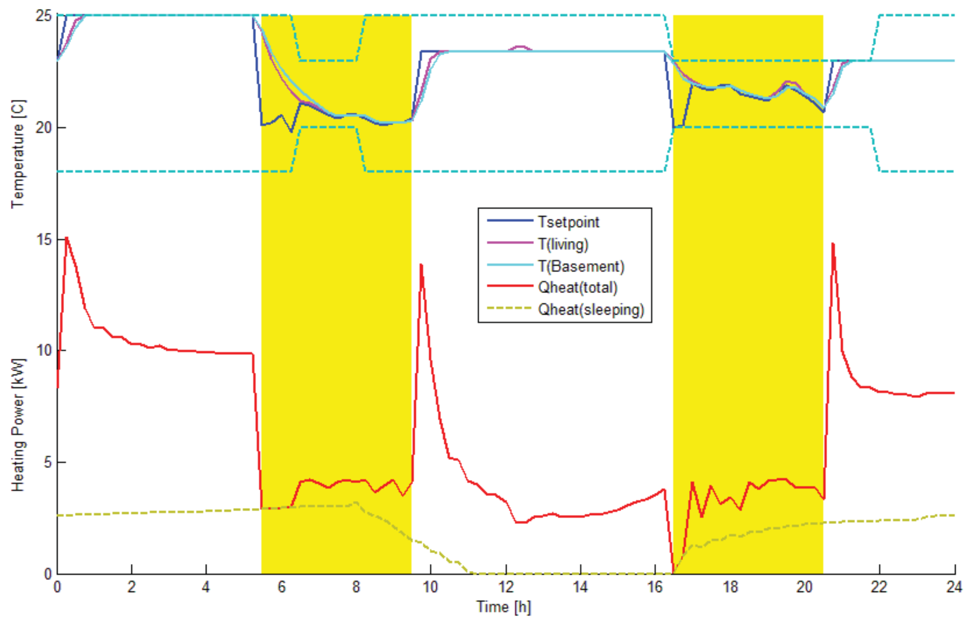

Figure 13: Minimum Power Demand at Peak Time

6. DISCUSSION

Table 1 summarizes all the approaches mentioned above. Power cost means the final value of cost function in each optimization and the reduction percentage is based on the "constant set point" case. Average and maximum power demand indicate the mean and maximum values of the heating power during the on-peak periods.

The GenOpt-TRNSYS MPC results in the lowest cost. Significant cost savings are obtained from the online Matlab (RC-model) MPC. The latter implies considerably lower computational efforts compared to the GenOpt-TRNSYS approach, as depicted by the Indicative CPU Time. The Exponential profile shows near-optimal results while its computational time is much less than the "full" GenOpt-TRNSYS MPC. Jump and trim results in savings very close to the exponential profile. The GenOpt-TRNSYS MPC has the minimum electricity cost while the Min Power Demand method has the lowest power demand, which is just over 4kW.

Table 1: Summary of Results

	Power Cost [-]	Power Cost Reduction [%]	Average Power Demand (peak) [kW]	Max Power Demand (peak) [kW]	Indicative CPU Time[-]*
Constant	51.4	0.0%	6.0	8.1	16 sec
Night Setback	56.3	-9.5%	6.6	15.9	19 sec
MPC with GenOpt	23.2	54.9%	2.6	5.6	16.3 h
MPC with Matlab	34.5	32.8%	3.9	6.8	4 min
Jump and trim	26.4	48.6%	3.0	4.9	18 min
Linear	31.9	37.9%	3.6	7.7	25 min
Exponential	26.0	49.4%	2.9	5.4	21 min
Min Power Demand	31.2	39.3%	3.5	4.2	6.7 h

* Indicative CPU Time is the time taken to run a 12-day TRNSYS simulation including the GenOpt or Matlab optimization process (only one day is optimized).The figures above show that the heating power profile during off-peak is affected by the different strategies, with large peaks at the beginning of

preheating periods. These large peaks would also be present with more conventional setbacks strategies, but not with a constant setpoint. The impact on overall system efficiency, capital or maintenance cost is probably negligible for electric baseboard heating, as considered in this study. However, this impact needs to be taken into account if other heating system types were considered (e.g. hydraulic heating with boiler or heat pump).

6.1 Limitations of the present study

This study considers the performance of different control strategies for the coldest day of the simulated period, and measures that performance by the average power use during on-peak periods. This is in line with the paper objective, which is to compare different model structures and to adopt the point of view of a utility trying to assess potential strategies to be implemented in a large number of individual houses. It could be used to deal with critical peak demand events that typically occur a few times per year. Different contexts would obviously require a different cost function and might require a performance comparison on a longer period.

The selected house is a relatively lightweight building, and results would be different for very light or heavy buildings.

Two main simplifications were made in this study in developing predictive control strategies. First, perfect forecasting was assumed for internal gains, occupancy and weather. Second, the model used to assess the results is the same model as the one used internally by the GenOpt-TRNSYS approach. The Matlab (RC-model) approach on the other hand used a simplified model internally but was assessed with the detailed TRNSYS model, which can be seen as more realistic, as all models will likely have errors compared to a real building. Results need to be confirmed using realistic forecasts and even more detailed models for the assessment or experimental validation.

The obtained profiles are sensitive to optimization parameters (cost function and initial values) and to numerical noise. The unintuitive shape of some results, such as the lack of pre-heating before the afternoon on-peak period in some of them, seems to indicate that the optimization results may still be quite far from the actual optimum. With this in perspective, the simplified (heuristics-based) GenOpt-TRNSYS methods definitely seem to be more interesting than the "brute force" approach.

Further work will address these points, first by investigating the sensitivity of optimization methods and alternative implementations, then by implementing the developed algorithms in real buildings.

7. CONCLUSIONS

Two different approaches were compared to optimize the power usage of electric heating during critical peak events in a typical Canadian house. Perfect forecasts were assumed in both cases. Both methods were tested in TRNSYS using a detailed building model. One method (GenOpt-TRNSYS) used the same model internally to perform the optimization, while the second method relied on a much simpler (RC) model implemented in Matlab.

Both methods deliver a significant reduction in power usage during the on-peak periods. The GenOpt-TRNSYS method delivers the largest savings (more than 50 %) at the cost of a very high computational effort (simulation time increased by a factor of over 3000 compared to the base case). The RC-model approach delivers less impressive savings (33 %) but at a much more reasonable computational cost (simulation time increased by a factor 12 compared to the base case). Heuristic methods restricting the possible setpoint profiles to predefined shapes were shown to deliver near-optimal savings with significantly reduced computational power (simulation time increased by a factor of 60 to 80 compared to the base case).

Both methods are sensitive to optimization parameters and results need to be confirmed on a wider set of test cases to assess the optimality of obtained results and their robustness. Realistic forecasts should also be introduced prior to implementation into real buildings in order to validate the proposed methodology.

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REFERENCES

Braun, J. E.-H. (2006). Assessment of demand limiting using building thermal mass in small commercial buildings. *ASHRAE Transactions*, 112 PART 1, 547-558.

Candanedo, J., Dehkordi, V., & Lopez, P. (2014). A multi-level architecture to facilitate MPC implementation in commercial buildings: basic principles and case study. *Proceedings of eSim 2014 conference.*

Coffey, E. B. (2013). Approximating model predictive control with existing building simulation tools and offline optimization. *Journal of Building Performance Simulation, 6*(3), 220-235.

CSTB. (2012). *Règlementation Thermique 2012 - Règles d'application Th-Bât.* CSTB Edition.

Henze, G., Felsmann, C., & Knabe, G. (2004). Evaluation of optimal control for active and passive building thermal storage. *International Journal of Thermal Sciences, 43*(2), 173–183.

Henze, G., Le, T., Florita, A., & Felsmann, C. (2006). Sensitivity Analysis of Optimal Building Thermal Mass Control. *Journal of Solar Energy Engineering, 129*(4), 473-485.

Huchuk, B., A. Cruickshank, C., O'Brien, W., & Gunay, H. (2014). Recursive thermal building model training using Ensemble. *Proceedings of eSim 2014 conference.*

Hydro-Québec. (2014, 01 22). *Hydro-Québec demande à ses clients de poursuivre leurs efforts de réduction de la consommation d'électricité en période de pointe et les remercie de leur collaboration.* Retrieved August 01, 2014, from http://cnw.ca/ONjV5

IBPSA-Canada. (2011). *Model predictive control in buildings workshop.* Retrieved September 10, 2014, from http://www.ibpsa.us/conferences

ISO-1. (2008). ISO 10211:2008 - Thermal bridges in building construction - Heat flows and surface temperatures - Detailed calculations.

ISO-2. (2008, June). ISO 10456: 2008 - Building materials and products - Hygrothermal properties - Tabulated design values and procedures for determining declared and design thermal values.

ISO-3. (2008, April). ISO 13370: 2008 - Thermal performance of buildings - Heat transfer via the groung - Calculation methods.

ISO-4. (2013). ISO 13790: 2013 - Energy performance of buildings - Calculation of energy use for space heating and cooling. *Energy performance of buildings - Calculation of energy use for space heating and cooling*.

ISO-5. (2008). ISO 6946:2008 - Building components and building elements - Thermal resistance and thermal transmittance - Calculation method.

Kelly, G. (1988). Control system simulation in North America. *Energy and Buildings, 10*(3), 193–202.

Kummert, M., Leduc , M.-A., & Moreau , A. (2011). Using MPC to reduce the peak demand associated with electric heating. *Model Predictive Control in Buildings Workshop.* Montréal.

Lehmann, B., Gyalistras, D., Gwerder, M., Wirth, K., & Carl, S. (2013). Intermediate complexity model for Model Predictive Control of Integrated Room. *Energy and Buildings*(58), 250–262.

Ma, Y., & Borrelli, F. (2011). Hierarchical Predictive Control Energy Efficient Buildings. *Model Predictive Control in Buildings Workshop.* Montreal.

Madson, H., & Holst, J. (1995). Estimation of continuous-time models for the heat dynamics. *Energy and Buildings*, 67-79.

May-Ostendorp, P., Henze, G., Corbin, C., Rajagopalan, B., & Felsmann, C. (2011). Model-predictive control of mixed-mode buildings with rule extraction. *Building and Environment, 46*(2), 428–437.

May-Ostendorp, P., Henze, G., Rajagopalan, B., & Corbin, C. (2013). Extraction of supervisory building control rules from model predictive control of windows in a mixed mode building. *Journal of Building Performance Simulation, 6*(3), 199-219.

Morari, M., & Lee, J. (1999). Model predictive control: past, present and future. *Computers & Chemical Engineering*, 667–682.

Nassif, N., Kajl, S., & Sabourin, R. (2005a). Simplified model-based optimal control of VAV air conditioning system. Montreal: Proceedings of the 9th international building performance simulation association (IBPSA) conference.

Nassif, N., Stainslaw , K., & Sabourin, R. (2005b). Optimization of HVAC control system strategy using two-objective genetic algorithm. *International Journal of HVAC&R Research, 11*(3), 459-486.

Oldewurtel, F., Parisio, A., Jones, C., & Morari, M. (2010). Energy efficient building climate control using Stochastic Model Predictive Control and weather predictions. *American Control Conference (ACC)*, (pp. 5100 - 5105). Baltimore, MD .

Oldewurtel, F., Parisio, A., Jonesc, C. N., Gyalistrasa, D., Gwerderd, M., Stauche, V., . . . Moraria, M. (2012). Use of model predictive control and weather forecasts for energy efficient building climate control. *Energy and Buildings* (45), 15–27.

Prívara, S., Cigler, J., Váňa, Z., Oldewurtel, F., Sagerschnig, C., & Žáčeková, E. (2013). Building modeling as a crucial part for building predictive control. *Energy and Buildings, 56*, 8–22.

Sturzenegger, D., Gyalistras, D., Semeraro, V., Morari, M., & Smith, R. S. (2014). BRCM Matlab Toolbox: Model Generation for Model Predictive Building Control. *American Control Conference.*

Swinton, M. C., Moussa, H., & Marchand, R. J. (2001). Commissioning Twin Houses for Assessing the Performance of Energy-Conserving Technologies. Thermal Performance of the Exterior Envelopes of Whole Buildings VIII.

Verhelst, C., Sourbron, M., Antonov, S., & Helsen , L. (2011). Towards MPC for office buildings with TABS connected to a GCHP system controller model. *Model Predictive Control in Buildings Workshop.* Montreal.

Wang , S., & Xu, X. (2006). Simplified building model for transient thermal performance estimation. *International Journal of Thermal Sciences*(45), 419–432.

Wang, S., & Jin, X. (2000). Model-based optimal control of VAV air-conditioning system using genetic algorithm. *Building and Environment, 35*(6), 471–487.

Wetter, M. (2001). GenOpt® -- A Generic Optimization Program. Proceedings of the 7th IBPSA Conference. Retrieved from http://gundog.lbl.gov/GO/

Wetter, M. (2011). *GenOpt, Generic Optimization Program, User Manual Version 3.1.0.* Retrieved July 01, 2014, from http://gundog.lbl.gov/GO/

Wystrcil, e. a. (2013). Model-based optimization of control strategies for low-exergy space heating systems using an environmental heat source. *13th Conference of International Building Performance Simulation Association.* Chambéry: BS2013.